



# Energy levels, transition dipole moment, transition probabilities and radiative lifetimes for low-lying electronic states of PN

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## ABSTRACT

The valence internally contracted multireference configuration-interaction (icMRCI) method is used to compute potential energy curves (PECs) of the  $X^1\Sigma^+$ ,  $A^1\Pi$ ,  $C^1\Sigma^-$ ,  $D^1\Delta$ ,  $2^1\Pi$ ,  $a^3\Sigma^+$ ,  $b^3\Pi$ ,  $d^3\Delta$ ,  $e^3\Sigma^-$ ,  $2^3\Delta$ ,  $2^3\Sigma^-$ ,  $1^5\Sigma^+$  and  $1^5\Pi$  states for PN, together with the Davidson, core-valence (CV) and scalar relativistic corrections, as well as the basis-set extrapolation. Transition dipole moments (TDMs) of fifteen dipole-allowed transitions between the thirteen states are calculated by the icMRCI approach with the aug-cc-pV6Z basis set. The vibrational band origins, Einstein coefficients and Franck-Condon factors of all spontaneous emissions for the fifteen band systems are determined, seeking to theoretically predict the strong emissions at least of the order of  $10^3 \text{ s}^{-1}$  for Einstein coefficients. Comparing with experimental measurements, our calculations can well reproduce the band origins and Franck-Condon factors of the  $A^1\Pi-X^1\Sigma^+$  system. Similar accuracy is assumed for the other band systems. Many emissions for the  $A^1\Pi-X^1\Sigma^+$ ,  $2^1\Pi-A^1\Pi$ ,  $2^1\Pi-X^1\Sigma^+$ ,  $2^1\Pi-C^1\Sigma^-$ ,  $2^1\Pi-D^1\Delta$ ,  $b^3\Pi-a^3\Sigma^+$ ,  $e^3\Sigma^-$ ,  $b^3\Pi$ ,  $2^3\Delta-1^3\Delta$ ,  $2^3\Sigma^-$ ,  $1^3\Sigma^-$ ,  $2^3\Sigma^-$ ,  $b^3\Pi$  and  $1^5\Pi-1^5\Sigma^+$  systems are found to be strong according to our calculated Einstein coefficients, whereas the emissions are weak for the  $2^3\Delta-b^3\Pi$  system. Radiative lifetimes for the first 15 vibrational levels are evaluated to be about tens of nanoseconds for the  $2^1\Pi$  state, about several hundred nanoseconds for the  $A^1\Pi$  state, about several to tens of microseconds for the  $b^3\Pi$ ,  $2^3\Delta$ ,  $2^3\Sigma^-$  and  $1^5\Pi$  states and about several to several hundred microseconds for the  $e^3\Sigma^-$  state. The results can be used as guidelines for line identification and diagnostics of astrophysical plasma.

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## 1. Introduction

Phosphorus mononitride (PN) is the first P-bearing molecule detected in the interstellar medium [1,2]. Subsequently, it was observed in the Origin "plateau" source [3], star-forming regions and cold cloud cores [4]. Over the past ten years or so, PN and other P-bearing species have been discovered by many research groups. These discoveries of P-bearing species, in the oxygen-rich supergiant star VY Canis Majoris [5,6], in the wind of the oxygen-rich AGB star IK Tauri [7], in massive dense cores [8–10], in solar-type star-forming regions [11] and in five molecular clouds located in the Galactic Center [12], greatly enhance our understanding of the interstellar medium, since many emission lines have been observed. It is well known that spectral lines can provide valuable information on astrophysical plasma. Analyzing the observed spectra requires knowledge of the transition properties of PN. However,

transition properties between triplet or quintet states of PN have not been investigated experimentally and theoretically.

Experimentally, spectroscopic observation of PN molecule was first reported by Curry et al. [13] for the bands in the region 2400–2900 Å. Analysis of the measured 24 bands gave an assignment of this system as a  $1^1\Pi-1^1\Sigma$  transition [14,15]. However, no attention had been paid to PN molecule until 1971 when Raymonda and Klemperer [16] provided a set of complementary structure parameters of PN by analyzing the molecular beam electric resonance spectrum. Since then, a number of experimental studies [17–29] have been performed and lots of spectral bands have been recorded. Most of them are designed to investigate the molecular structure and spectroscopic parameters of PN. In particular, Moeller and Silvers [17] obtained a table of relative transition probabilities of the  $A^1\Pi-X^1\Sigma^+$  system from the observed fluorescence spectra of PN. Ghosh et al. [20] reported a high resolution study of the  $A^1\Pi-X^1\Sigma^+$  system in the spectral range of 2200–3100 Å, including  $\nu' = 0-10$  to  $\nu'' = 0-11$  transitions. Ahmad and Hamilton [24] reported accurate infrared and microwave results of the  $A^1\Pi-X^1\Sigma^+$  system. Coquart and Prudhomme [18,19] first observed the absorption spectra of the  $E^1\Sigma^+-X^1\Sigma^+$  transition in the 1600–

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1900 Å region and found 11 vibrational levels of the new excited  $E^1\Sigma^+$  state. Verma et al. [22] identified some absorption bands in the spectral region 1300–1500 Å as the  $\alpha^1\Pi-X^1\Sigma^+$ ,  $\beta^1\Pi-X^1\Sigma^+$ ,  $\gamma^1\Sigma^+-X^1\Sigma^+$  and  $\delta^1\Sigma^+-X^1\Sigma^+$  transitions by rotational analysis of these bands. Saraswathy and Krishnamurty [21] carried out a rotational analysis of 11 bands of the  $A^1\Pi-X^1\Sigma^+$  system for  $P^{14}N$  and 16 bands of the  $A^1\Pi-X^1\Sigma^+$  system for  $P^{15}N$ , along with the perturbation in the  $v=0-4$  levels of  $A^1\Pi$  state caused by the  $e^3\Sigma^-$ ,  $d^3\Delta$  and  $b^3\Pi$  states. Such study was improved later by Floch et al. [26]. In addition, Bredohl et al. [23] observed the absorption spectrum of PN up to 1050 Å and determined 5 Rydberg series.

Theoretically, a great deal of work [30–44] has been done on the electronic structure, potential energy and spectroscopic parameters of PN. Among these theoretical studies, only one group investigated the transition properties of PN, i.e., de Brouckere et al. [35,37] determined the pure rotational, rovibrational transitions and spontaneous radiative lifetimes for the  $X^1\Sigma^+$  ground state of PN and calculated the Einstein coefficients and Franck-Condon factors of the  $A^1\Pi-X^1\Sigma^+$  and  $D^1\Delta-A^1\Pi$  systems.

Analyzing previous experimental results, we can find that many spectral bands have been recorded for the  $A^1\Pi-X^1\Sigma^+$  system of PN, together with numerous vibrational band origins. Four triplet  $a^3\Sigma^+$ ,  $e^3\Sigma^-$ ,  $d^3\Delta$  and  $b^3\Pi$  states have been observed by analysis of their perturbation in the  $A^1\Pi$  state, but little spectroscopic data of them have been obtained. Experimental observation seems not to have reported the transition properties between triplet or quintet states over the past few decades. We wonder whether the transitions between triplet or quintet states are too weak to observe. Moreover, theoretical studies have not been carried out for radiative transition probabilities between triplet or quintet states. Radiative lifetimes of any triplet or quintet states are not available in the literature. So the transition properties of PN are worthy of further investigation.

This work is organized as follows. In Section 2, we briefly present the theory and method, including the dissociation relationships between the electronic states and the dissociation limits of PN, the approaches of calculating the potential energy curves (PECs) and the transition dipole moments (TDMs), and the formulas of computing the transition parameters. In Section 3, PECs and TDMs are reported. The vibrational band origins, Einstein coefficients of spontaneous emission (hereafter called Einstein coefficients) and Franck-Condon factors are determined for dipole-allowed transition systems. Radiative lifetimes are also evaluated for the  $A^1\Pi$ ,  $D^1\Delta$ ,  $2^1\Pi$ ,  $b^3\Pi$ ,  $e^3\Sigma^-$ ,  $2^3\Delta$ ,  $2^3\Sigma^-$  and  $1^5\Pi$  states. In Section 4, conclusions are drawn. These results can be used as guidelines for line identification and diagnostics of astrophysical plasma.

## 2. Theory and method

### 2.1. Dissociation relationships

The ground states of nitrogen (N) and Phosphorus (P) are  $(2p^3)^4S_u$  and  $(3p^3)^4S_u$ , respectively. The first and second excited states of N are  $(2p^3)^2D_u$  and  $(2p^3)^2P_u$ , respectively. Their energy levels relative to the ground state are 19,228.82 cm<sup>-1</sup> and 28,839.11 cm<sup>-1</sup> [45], which are obtained, respectively, by averaging the energy of the  $(2p^3)^2D_{3/2}$  and  $(2p^3)^2D_{5/2}$  states and by averaging the energy of the  $(2p^3)^2P_{1/2}$  and  $(2p^3)^2P_{3/2}$  states. The first and second excited states of P are  $(3p^3)^2D_u$  and  $(3p^3)^2P_u$ , respectively. Their energy levels relative to the ground state are 11,368.83 cm<sup>-1</sup> and 18,735.36 cm<sup>-1</sup> [45], which are obtained, respectively, by averaging the energy of the  $(3p^3)^2D_{1/2}$  and  $(3p^3)^2D_{3/2}$  states and by averaging the energy of the  $(3p^3)^2P_{1/2}$  and  $(3p^3)^2P_{3/2}$  states. Therefore, the first six dissociation limits of PN are  $P(^4S_u)+N(^4S_u)$ ,

$P(^4D_u)+N(^4S_u)$ ,  $P(^2P_u)+N(^4S_u)$ ,  $P(^4S_u)+N(^2D_u)$ ,  $P(^4S_u)+N(^2P_u)$  and  $P(^2D_u)+N(^2D_u)$ , respectively. These six dissociation limits and their generating 54 electronic states are presented in Table 1. The energy separations between each higher dissociation limit and the lowest one are calculated and given in Table 1, which are in excellent agreement with the experimental data [45].

### 2.2. PECs and TDMs

Using the complete active space self-consistent field (CASSCF) wave functions [46] as reference, PECs are calculated by the internally contracted multireference configuration-interaction (icMRCI) method [47,48] with Davidson correction (+Q). These calculations are done in  $C_{2v}$  symmetry and performed in the MOLPRO 2015 program suite [49,50]. In the calculations, nine outermost molecular orbitals (MOs) ( $5a_1$ ,  $2b_1$  and  $2b_2$ ), i.e.,  $5-9\sigma$ ,  $2\pi$  and  $3\pi$  MOs, are chosen as active orbitals. 10 valence electrons of PN are distributed into 9 valence MOs, the other 12 inner electrons are put into 6 closed-shell MOs ( $4a_1$ ,  $1b_1$  and  $1b_2$ ), i.e.,  $1-4\sigma$  and  $1\pi$  MOs.

To obtain more accurate PECs, potential energy is extrapolated to the complete basis set (CBS) limit with the aug-cc-pV5Z (AV5Z) and aug-cc-pV6Z (AV6Z) basis sets of Dunning [38,51,52], denoted as “56”. Extrapolation formulas used here are given by Truhlar [53]

$$E_X^{ref} = E_\infty^{ref} + A^{ref} X^{-\alpha} \quad (1)$$

$$E_X^{cor} = E_\infty^{cor} + A^{cor} X^{-\beta} \quad (2)$$

where  $E_X^{ref}$  and  $E_X^{cor}$  are the reference and correlation energies, respectively, computed by the AVXZ basis set (here  $X=5$  and 6).  $E_\infty^{ref}$  and  $E_\infty^{cor}$  are the reference and correlation energies, respectively, determined by the CBS extrapolation. Extrapolation parameters  $\alpha$  and  $\beta$  are given by Truhlar [53]. Core valence (CV) correlation energy correction is obtained by the icMRCI technique using the cc-pCV5Z basis set [38]. Scalar relativistic energy correction is calculated via third-order Douglas-Kroll-Hess (DKH3) Hamiltonian approximation [54,55] at the icMRCI/cc-pV5Z-DK [56] level of theory. Finally, PECs are obtained by the icMRCI+Q/56+CV+DK method.

TDMs are calculated by the valence icMRCI approach with the AV6Z basis set, as implemented in the MOLPRO 2015 program suite. The obtained PECs and TDMs are used to compute the radiative transition probabilities for dipole-allowed transitions of PN. Radiative lifetimes of vibrational levels for some electronic states of PN are also evaluated. Such transition parameters will be elaborated below.

### 2.3. Transition probabilities

Transition probabilities such as Einstein coefficients  $A_{v'v''}$  and Franck-Condon factors  $q_{v'v''}$  between an initial vibrational level  $v'$  in the upper electronic state and a final vibrational level  $v''$  in the lower electronic state are as follows [57,58]

$$A_{v'v''} = 2.026 \times 10^{-6} \sigma_{v'v''}^3 \frac{2 - \delta_{0,\Lambda'+\Lambda''}}{2 - \delta_{0,\Lambda'}} (R_e^{v'v''})^2 \quad (3)$$

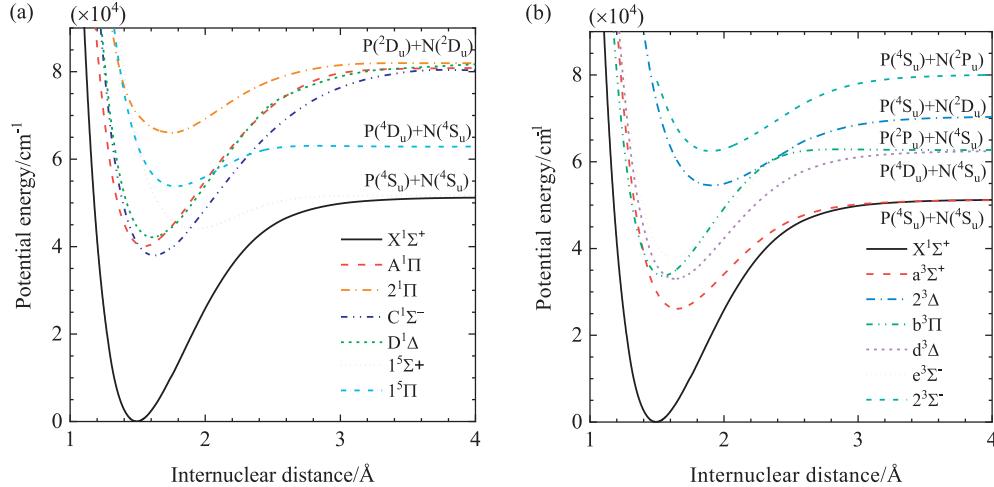
$$q_{v'v''} = \left( \int \psi_{v'}(r) \psi_{v''}(r) dr \right)^2 \quad (4)$$

where  $\sigma_{v'v''}$  is the wavenumber of a transition system,  $\Lambda'$  and  $\Lambda''$  are the projections of electronic orbital angular momentum on the internuclear axis for the upper and lower electronic levels, respectively.  $\psi_{v'}(r)$  and  $\psi_{v''}(r)$  are the vibrational wave functions in the upper electronic state and those in the lower electronic state, respectively.  $(R_e^{v'v'})$  is the square of electronic-vibrational transition

**Table 1**

Dissociation relationships of the 54 electronic states generated from the first six dissociation limits of PN.

Dissociation limit	Electronic state	Relative energy (cm <sup>-1</sup> )	
		This work <sup>a</sup>	Expt. [45]
P( <sup>4</sup> S <sub>u</sub> ) + N( <sup>4</sup> S <sub>u</sub> )	X <sup>1</sup> Σ <sup>+</sup> , a <sup>3</sup> Σ <sup>+</sup> , 1 <sup>5</sup> Σ <sup>+</sup> , 1 <sup>7</sup> Σ <sup>+</sup>	0.00	0.00
P( <sup>4</sup> D <sub>u</sub> ) + N( <sup>4</sup> S <sub>u</sub> )	2 <sup>3</sup> Σ <sup>+</sup> , b <sup>3</sup> Π, d <sup>3</sup> Δ, 2 <sup>5</sup> Σ <sup>+</sup> , 1 <sup>5</sup> Π, 1 <sup>5</sup> Δ	11,415.49	11,368.83
P( <sup>2</sup> P <sub>u</sub> ) + N( <sup>4</sup> S <sub>u</sub> )	e <sup>3</sup> Σ <sup>-</sup> , 2 <sup>3</sup> Π, 1 <sup>5</sup> Σ <sup>-</sup> , 2 <sup>5</sup> Π	18,577.95	18,735.36
P( <sup>4</sup> S <sub>u</sub> ) + N( <sup>2</sup> D <sub>u</sub> )	3 <sup>3</sup> Σ <sup>+</sup> , 3 <sup>3</sup> Π, 2 <sup>3</sup> Δ, 3 <sup>5</sup> Σ <sup>+</sup> , 3 <sup>5</sup> Π, 2 <sup>5</sup> Δ	19,097.32	19,228.82
P( <sup>4</sup> S <sub>u</sub> ) + N( <sup>2</sup> P <sub>u</sub> )	2 <sup>3</sup> Σ <sup>-</sup> , 4 <sup>3</sup> Π, 2 <sup>5</sup> Σ <sup>-</sup> , 4 <sup>5</sup> Π	28,780.23	28,839.11
P( <sup>2</sup> D <sub>u</sub> ) + N( <sup>2</sup> D <sub>u</sub> )	2 <sup>1</sup> Σ <sup>+</sup> , 3 <sup>1</sup> Σ <sup>+</sup> , 4 <sup>1</sup> Σ <sup>+</sup> , C <sup>1</sup> Σ <sup>-</sup> , 2 <sup>1</sup> Σ <sup>-</sup> , A <sup>1</sup> Π, 2 <sup>1</sup> Π, 3 <sup>1</sup> Π, 4 <sup>1</sup> Π, D <sup>1</sup> Δ, 2 <sup>1</sup> Δ, 3 <sup>1</sup> Δ, 1 <sup>1</sup> Φ, 2 <sup>1</sup> Φ, 1 <sup>1</sup> Γ, 2 <sup>3</sup> Σ <sup>+</sup> , 3 <sup>3</sup> Σ <sup>+</sup> , 4 <sup>3</sup> Σ <sup>+</sup> , C <sup>3</sup> Σ <sup>-</sup> , 2 <sup>3</sup> Σ <sup>-</sup> , A <sup>3</sup> Π, 2 <sup>3</sup> Π, 3 <sup>3</sup> Π, 4 <sup>3</sup> Π, D <sup>3</sup> Δ, 2 <sup>3</sup> Δ, 3 <sup>3</sup> Δ, 1 <sup>3</sup> Φ, 2 <sup>3</sup> Φ, 1 <sup>3</sup> Γ	30,174.01	30,104.19

<sup>a</sup> Obtained by the icMRCI+Q/56+CV+DK calculations.**Fig. 1.** PECs of (a) the X<sup>1</sup>Σ<sup>+</sup>, A<sup>1</sup>Π, C<sup>1</sup>Σ<sup>-</sup>, D<sup>1</sup>Δ, 2<sup>1</sup>Π, 1<sup>5</sup>Σ<sup>+</sup> and 1<sup>5</sup>Π electronic states (b) the a<sup>3</sup>Σ<sup>+</sup>, b<sup>3</sup>Π, d<sup>3</sup>Δ, e<sup>3</sup>Σ<sup>-</sup>, 2<sup>3</sup>Δ and 2<sup>3</sup>Σ<sup>-</sup> electronic states for PN calculated at the icMRCI+Q/56+CV+DK level of theory. The PECs are presented relative to the minimum of the ground state.

moment, which can be expressed in terms of wave function and TDM [58]

$$(R_e^{v'v''})^2 = \left[ \int_0^\infty \psi_{v'}(r) R_e(r) \psi_{v''}(r) dr \right]^2 \quad (5)$$

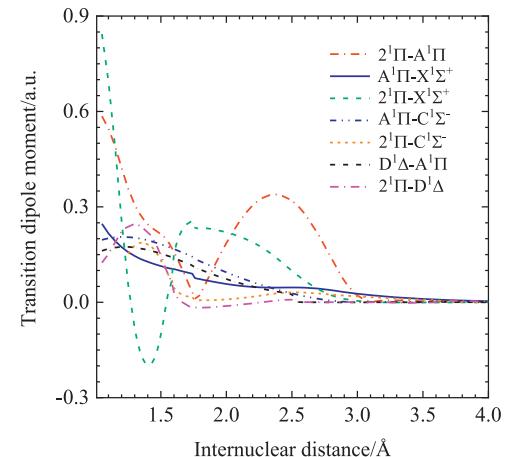
where  $R_e(r)$  is the TDM. The total Einstein coefficients of a certain vibrational level  $v'$  in the upper electronic state are obtained by summing the Einstein coefficients from this level to all possible vibrational levels  $v''$  in the lower electronic state. The radiative lifetimes of this level can be obtained by the reciprocal of the total Einstein coefficients

$$\tau_{v'} = \frac{1}{\sum_{v''=0}^{v'_{\max}} A_{v'v''}} \quad (6)$$

where  $\tau_{v'}$  is radiative lifetimes of vibrational level  $v'$ .

### 3. Results and discussion

The PECs of the X<sup>1</sup>Σ<sup>+</sup>, A<sup>1</sup>Π, C<sup>1</sup>Σ<sup>-</sup>, D<sup>1</sup>Δ, 2<sup>1</sup>Π, a<sup>3</sup>Σ<sup>+</sup>, b<sup>3</sup>Π, d<sup>3</sup>Δ, e<sup>3</sup>Σ<sup>-</sup>, 2<sup>3</sup>Δ, 2<sup>3</sup>Σ<sup>-</sup>, 1<sup>5</sup>Σ<sup>+</sup> and 1<sup>5</sup>Π electronic states for PN are calculated at the icMRCI+Q/56+CV+DK level of theory and shown in Fig. 1. Figs. 2 and 3 present the TDMs of 15 band transition systems. The obtained PECs are involved in the LEVEL program [59] to calculate the spectroscopic parameters (Table 2). Specifically speaking, the vibrational levels  $G_v$  and internal rotation constant  $B_v$  are first determined by solving the nuclear radial Schrödinger equation over the PECs. The spectroscopic parameters of each state are then fitted by its  $G_v$  and  $B_v$  values, which are given in the Appendix for determining the vibrational band origins. The spectral regions of strong emissions for 15 band systems are presented in Table 3.

**Fig. 2.** Curves of TDM versus internuclear distance for seven band transition systems between five singlet states.

In order to clearly understand the electronic transitions between different states, the dominant valence electron configurations of each state near the equilibrium internuclear distance are calculated at the icMRCI/AV6Z level of theory and listed in Table 4. Using the PECs and TDMs obtained here, we compute the radiative transition probabilities for 15 dipole allowed transitions. Some relatively large Einstein coefficients and Franck-Condon factors are listed in Tables 5–10. The radiative lifetimes of some excited states are also determined from the obtained Einstein coefficients and are given in Table 11. From the selection rules of electric dipole tran-









observed fluorescence and lamp intensities. Our calculated Franck-Condon factors of the 0–0, 0–1, 0–2, 0–3, 1–0, 1–1, 1–2, 1–3 and 1–4 bands are 0.5577, 0.3179, 0.0899, 0.0163, 0.3105, 0.0995, 0.3287, 0.1847 and 0.0509, which agree well with the semi-empirically calculated data of 0.58, 0.32, 0.09, 0.01, 0.31, 0.11, 0.34, 0.18 and 0.06, respectively [17]. In addition, the obtained Franck-Condon factors involving  $v' = 0\text{--}5$  to  $v'' = 0\text{--}4$  transitions compare favourably with those calculated by de Brouckere et al. [37], e.g., some relatively large Franck-Condon factors corresponding to the 0–0, 0–1, 1–0, 1–2 and 1–3 bands in this work are 0.5577, 0.3179, 0.3105 and 0.3287, respectively. Correspondingly, the theoretical results [37] of these bands are 0.5425, 0.3380, 0.3704, 0.3062, respectively.

The  $A^1\Pi$  state can also decay to the  $C^1\Sigma^-$  state. Table 2 presents the spectroscopic parameters of this state, which are in good agreement with previous theoretical ones [31]. Only Floch et al. [26] observed the  $C^1\Sigma^-$  state in the study of the perturbations in the  $A^1\Pi$  state. No spectroscopic information was obtained except for  $T = 43,048 \text{ cm}^{-1}$  and  $B = 0.6003 \text{ cm}^{-1}$  of an unassigned vibrational level for this state. As depicted in Fig. 1, the small energy difference between the  $A^1\Pi$  and  $C^1\Sigma^-$  states means that the emissions from the  $A^1\Pi$  state to the  $C^1\Sigma^-$  state are not very strong. Moreover, the TDMs of this system (Fig. 2) are not large enough. Hence, the obtained transition probabilities are not large enough, some of which are collected in Table 5.

Using the calculated Einstein coefficients of the  $A^1\Pi-X^1\Sigma^+$  and  $A^1\Pi-C^1\Sigma^-$  systems, we calculate the radiative lifetimes for the first 15 vibrational levels of the  $A^1\Pi$  state (Table 11). Due to the much larger Einstein coefficients of the  $A^1\Pi-X^1\Sigma^+$  system than those of the  $A^1\Pi-C^1\Sigma^-$  system, the radiative lifetimes of the  $A^1\Pi$  state are essentially determined by the  $A^1\Pi-X^1\Sigma^+$  transition. These radiative lifetimes of the  $A^1\Pi$  state are several hundred nanoseconds for all the calculated vibrational levels. Experimentally, Hanle signal detection [62] provided the radiative lifetime of  $227 \pm 70 \text{ ns}$  for  $v' = 0$  of  $A^1\Pi$  state at zero pressure. The theoretical radiative lifetime calculated by de Brouckere et al. [37] for  $v' = 0$  is 742.4 ns. Our calculated one is 695.4 ns, which is in good agreement with the theoretical one [37], but larger than the experimental value [62]. In fact, in practical measurements, the  $A^1\Pi$  state is most likely to be perturbed by the adjacent electronic states, especially the very nearby  $D^1\Delta$  state, which may decrease the radiative lifetime. de Brouckere et al. [37] also pointed out that radiative lifetime increases with the increasing vibrational quantum number. The opposite result is obtained here. Most probably it is because de Brouckere et al. [37] only consider the Einstein coefficients for  $(v', v'') = (0\text{--}4, 0\text{--}4)$ , whereas Einstein coefficients involving higher vibrational levels  $v'$  and  $v''$  are taken into account in this work.

The  $D^1\Delta$  state is dominated by the  $5\sigma^2 6\sigma^2 2\pi^3 7\sigma^2 3\pi^1 8\sigma^0$  electron configuration. The calculated spectroscopic parameters of the  $D^1\Delta$  state are listed in Table 2, which agree well with some theoretical results [31,37]. However, no definite spectroscopic parameters of the  $D^1\Delta$  state are obtained experimentally except for little information of the observed perturbation in the  $A^1\Pi$  state [20,26].

As shown in Fig. 2, The curve of the TDM for the  $D^1\Delta-A^1\Pi$  system is similar to that of the  $A^1\Pi-C^1\Sigma^-$  system along the internuclear distance, which is not too surprising as the  $D^1\Delta$  and  $C^1\Sigma^-$  states have similar molecular orbital compositions, differing only in the coupling of the angular moments. The  $R_e$  of the  $D^1\Delta$  state is very close to that of the  $A^1\Pi$  state and they both possess deep potential wells. Hence, many transitions for this system should exist. Small TDM values of this system also show that these transitions should not be strong. The number of the Einstein coefficients, which are of the order of  $10^3$  and  $10^2$ , is 147 and 429, respectively. Some of relatively large Einstein coefficients are presented in Table 5. In addition, radiative lifetime is relatively large

for low-lying vibrational level of the  $D^1\Delta$  state. So the  $A^1\Pi-C^1\Sigma^-$  system can be observed, but with great efforts.

The  $2^1\Pi$  state is formed after promotions of two electrons from the  $2\pi 7\sigma$  MOs into the  $3\pi$  MO. This state can undergo dipole-allowed transitions to all of the lower singlet  $A^1\Pi$ ,  $X^1\Sigma^+$ ,  $C^1\Sigma^-$  and  $D^1\Delta$  states. As shown in Fig. 2, the curve of TDM for the  $2^1\Pi-A^1\Pi$  system takes up a large area relative to the  $R_e$ -axis. So many intense emissions should exit for this system. Table 5 presents some relatively large Einstein coefficients and Franck-Condon factors for this system. Due to the large energy separation (Fig. 1) and large TDMs (Fig. 2) between  $2^1\Pi$  and  $X^1\Sigma^+$  states, there are a great deal of intense emissions for the  $2^1\Pi-X^1\Sigma^+$  system. For the  $2^1\Pi-C^1\Sigma^-$  and  $2^1\Pi-D^1\Delta$  systems, the curves of TDM are similar, with relatively large values in the Franck-Condon region. Large Einstein coefficients are obtained near diagonal matrix elements for these two systems.

Overall, the  $A^1\Pi-X^1\Sigma^+$ ,  $2^1\Pi-A^1\Pi$ ,  $2^1\Pi-X^1\Sigma^+$ ,  $2^1\Pi-C^1\Sigma^-$  and  $2^1\Pi-D^1\Delta$  systems should not be difficult to measure by appropriate experiments. The  $A^1\Pi-C^1\Sigma^-$  and  $D^1\Delta-A^1\Pi$  systems can also be detected with some efforts. Among the transitions from  $2^1\Pi$  state to the lower  $A^1\Pi$ ,  $X^1\Sigma^+$ ,  $C^1\Sigma^-$  and  $D^1\Delta$  states, the emissions of the  $2^1\Pi-X^1\Sigma^+$  system are most intense, followed by those of the  $2^1\Pi-A^1\Pi$  system.

### 3.2. Radiative transitions between triplet states

Of the triplet states of PN, the  $d^3\Delta$ ,  $e^3\Sigma^-$  and  $b^3\Pi$  states were identified by Saraswathy and Krishnamurty [21] from observation of the perturbations in the  $v = 0\text{--}4$  levels of the  $A^1\Pi$  state. Spectroscopic parameters of the  $e^3\Sigma^-$  and  $b^3\Pi$  states were also fitted by the observed energy levels lying at  $v = 0\text{--}4$  levels of the  $A^1\Pi$  state. So explicit vibrational levels of the  $e^3\Sigma^-$  and  $b^3\Pi$  states were not obtained. In order to compare our results with the experimental data, the calculated vibrational energy levels of the  $e^3\Sigma^-$  and  $b^3\Pi$  states approximately lying at  $v=0\text{--}4$  levels of the  $A^1\Pi$  state are used to fit the spectroscopic parameters, which are given in Table 7, together with the experimental data [21]. Although the energy levels may be not consistent with the observed ones, relatively good agreement is observed for the  $e^3\Sigma^-$  and  $b^3\Pi$  states. The spectroscopic parameters of the  $d^3\Delta$  state are not given because the large discrepancy between the calculated spectroscopic parameters and the experimental ones [21]. It is worth noting that our calculated spectroscopic parameters of the  $d^3\Delta$  state agrees well with those calculated by Grein and Kapur [31] and those computed by Abbiche et al. [44]. Moreover, a larger active space does not improve the spectroscopic parameters of the  $d^3\Delta$  state. So new experiments are needed to clarify such difference.

As shown in Fig. 3, the TDMs of the  $b^3\Pi-a^3\Sigma^+$  and  $b^3\Pi-d^3\Delta$  systems are relatively large over a wide range of internuclear distance. Moreover, the  $R_e$  of the  $b^3\Pi$  state is close to that of the  $a^3\Sigma^+$  state and that of the  $d^3\Delta$  state. The energy separation between the  $b^3\Pi$  and  $a^3\Sigma^+$  states is large, whereas the  $T_e$  of the  $b^3\Pi$  state is slightly larger than that of the  $d^3\Delta$  state. Based on these results, Franck-Condon principle predicts that the emissions of the  $b^3\Pi-a^3\Sigma^+$  system should be stronger than those of the  $b^3\Pi-d^3\Delta$  system. Our calculated Einstein coefficients and Franck-Condon factors verify such prediction. Some large ones are listed in Table 8.

Due to the symmetry limitation, The  $e^3\Sigma^-$  state can only spontaneously decay to the  $b^3\Pi$  state. The  $e^3\Sigma^-$ - $b^3\Pi$  system has similar TDMs to those of the  $b^3\Pi-a^3\Sigma^+$  and  $b^3\Pi-d^3\Delta$  systems, which can be explained by the same electron configuration of the  $e^3\Sigma^-$  state as those of the  $a^3\Sigma^+$  and  $d^3\Delta$  states.

The  $2^3\Delta$  state can spontaneously decay to the  $b^3\Pi$  and  $d^3\Delta$  states. As seen in Fig. 3, the TDM of the  $2^3\Delta-b^3\Pi$  system is near zero in the Franck-Condon region, which is not surprising as the two states differ by two electronic orbitals: the electron con-

figuration of the  $2^3\Delta$  state is  $5\sigma^2 6\sigma^2 2\pi^2 7\sigma^2 3\pi^2 8\sigma^0$ , while the  $b^3\Pi$  state is dominantly described by the  $5\sigma^2 6\sigma^2 2\pi^4 7\sigma^1 3\pi^1 8\sigma^0$  electron configuration. There are only a few large Einstein coefficients which are of the order of  $10^2$ . In general, this system should be hard to observe. The emissions of the  $2^3\Delta-1^3\Delta$  system are strong. Some large Einstein coefficients and Franck-Condon factors are given in Table 9.

The  $2^3\Sigma^-$  state is located at  $62,436.04\text{ cm}^{-1}$ . The depth of the potential is  $17,798.26\text{ cm}^{-1}$ , which carries 44 vibrational levels. This state can decay radiatively to the lower  $1^3\Sigma^-$  and  $b^3\Pi$  states. The vibrational band origins show that the emissions of the  $2^3\Sigma^--1^3\Sigma^-$  system lie in the UV, visible and infrared regions. These emissions are relatively strong, but intense emissions between low-lying states mainly locate in the UV and visible regions. So it should not be difficult to observe the  $2^3\Sigma^--1^3\Sigma^-$  system by a suitable experiment. Similar to the  $2^3\Delta-b^3\Pi$  system, the  $2^3\Sigma^--b^3\Pi$  system corresponds to the  $2\pi^2 7\sigma^2 3\pi^2-2\pi^4 7\sigma^1 3\pi^1$  transitions. Large Einstein coefficients occur in the bands corresponding to high  $v'$  and low  $v''$  and they are in the UV band. Thus, The  $2^3\Sigma^--b^3\Pi$  system can be observed with great efforts.

In general, the  $b^3\Pi-a^3\Sigma^+$ ,  $e^3\Sigma^--b^3\Pi$ ,  $2^3\Delta-1^3\Delta$  and  $2^3\Sigma^--1^3\Sigma^-$  systems should not be difficult to observe via appropriate spectroscopic techniques. The  $2^3\Sigma^--b^3\Pi$  and  $b^3\Pi-d^3\Delta$  systems can also be measured with some efforts. Emissions of the  $2^3\Delta-b^3\Pi$  system, however, should be difficult to obtain experimentally.

### 3.3. Radiative transitions between quintet states

The  $1^5\Sigma^+$  state is predicted to lie at  $44,095.84\text{ cm}^{-1}$  with a potential well of  $7717.77\text{ cm}^{-1}$ , which supports 24 vibrational levels. The  $1^5\Pi$  state is found at  $53,760.81\text{ cm}^{-1}$ . It possesses a well depth of  $9303.92\text{ cm}^{-1}$ . At the icMRCI+Q/56+CV+DK level of theory, the calculated spectroscopic parameters of the  $1^5\Sigma^+$  and  $1^5\Pi$  states are given in Table 2 and match well with recent theoretical ones [44].

As shown in Table 4, both the  $1^5\Pi$  and  $1^5\Sigma^+$  states have single reference configuration in nature around their corresponding equilibrium internuclear distances. The leading electronic transition between the two states is  $2\pi^3\sigma^1-2\pi^2 7\sigma^2$  promotion. As displayed in Fig. 3, TDMs of the  $1^5\Pi-1^5\Sigma^+$  system are relatively large in the Franck-Condon region. According to the Franck-Condon principle, the emissions of this system are expected to be relatively strong. Our calculations confirm this expectation with the fact that there are 59 and 119 emissions whose Einstein coefficients are of the order of  $10^4$  and  $10^3$ , respectively. In addition, radiative lifetimes of all the vibrational levels for the  $1^5\Pi$  state are relatively small (Table 11). Considering the factors above, we infer that the  $1^5\Pi-1^5\Sigma^+$  system should not be difficult to detect by means of spectroscopy.

## 4. Conclusions

Utilizing the PECs calculated at the icMRCI+Q/56+CV+DK level of theory and the TDMs calculated by the icMRCI/AV6Z method, we have determined the vibrational band origins, Einstein coefficients and Franck-Condon factors for dipole-allowed transition systems between the  $X^1\Sigma^+$ ,  $A^1\Pi$ ,  $C^1\Sigma^-$ ,  $D^1\Delta$ ,  $2^1\Pi$ ,  $a^3\Sigma^+$ ,  $b^3\Pi$ ,  $d^3\Delta$ ,  $e^3\Sigma^-$ ,  $2^3\Delta$ ,  $2^3\Sigma^-$ ,  $1^5\Sigma^+$  and  $1^5\Pi$  states, and have evaluated the vibrational radiative lifetimes of the  $A^1\Pi$ ,  $D^1\Delta$ ,  $2^1\Pi$ ,  $b^3\Pi$ ,  $e^3\Sigma^-$ ,  $2^3\Delta$ ,  $2^3\Sigma^-$  and  $1^5\Pi$  states. Several main conclusions have been reached below.

- (1) The  $2^1\Pi-A^1\Pi$  system covers a wide spectral range from UV to far-infrared region. The  $2^1\Pi-X^1\Sigma^+$  system extends from vacuum UV to visible band. The  $2^3\Delta-D^1\Delta$  system spreads

from UV to mid-infrared wavelengths. The  $1^5\Pi-1^5\Sigma^+$  system mainly lies in the VUV region. The spectral range are in the UV band for the  $A^1\Pi-X^1\Sigma^+$ ,  $2^1\Pi-C^1\Sigma^-$ ,  $2^1\Pi-D^1\Delta$ ,  $2^3\Delta-b^3\Pi$  and  $2^3\Sigma^--b^3\Pi$  systems, and in the infrared region for the  $A^1\Pi-C^1\Sigma^-$ ,  $D^1\Delta-A^1\Pi$ ,  $b^3\Pi-a^3\Sigma^+$ ,  $e^3\Sigma^--b^3\Pi$  and  $b^3\Pi-d^3\Delta$  systems. All these spectral ranges can be used to provide information for measuring these systems in the experiments.

- (2) Spectral emissions of the  $A^1\Pi-X^1\Sigma^+$ ,  $2^1\Pi-A^1\Pi$ ,  $2^1\Pi-X^1\Sigma^+$ ,  $2^1\Pi-C^1\Sigma^-$ ,  $2^1\Pi-D^1\Delta$ ,  $b^3\Pi-a^3\Sigma^+$ ,  $e^3\Sigma^--b^3\Pi$ ,  $2^3\Delta-1^3\Delta$ ,  $2^3\Sigma^--1^3\Sigma^-$  and  $1^5\Pi-1^5\Sigma^+$  systems should not be difficult to observe experimentally. Spectral emissions of the  $A^1\Pi-C^1\Sigma^-$ ,  $D^1\Delta-A^1\Pi$ ,  $2^3\Sigma^--b^3\Pi$  and  $b^3\Pi-d^3\Delta$  systems can also be detected with some efforts. Whereas spectral emissions of the  $2^3\Delta-b^3\Pi$  system should be difficult to measure according to Franck-Condon principle.
- (3) Radiative lifetimes of the vibrational levels are about tens of nanoseconds for the  $2^1\Pi$  state, about several hundred nanoseconds for the  $A^1\Pi$  state and about several to tens of microseconds for the  $b^3\Pi$ ,  $2^3\Delta$ ,  $2^3\Sigma^-$  and  $1^5\Pi$  states. Such short radiative lifetimes mean that the spontaneous transitions from these states can easily occur. Radiative lifetimes of the vibrational levels for the  $e^3\Sigma^-$  state about several to several hundred microseconds. Radiative lifetimes are very long for the lower vibrational levels of the  $D^1\Delta$  state.
- (4) Large transition parameters of several band systems are calculated and presented in this work, which can help in line identification and diagnostics of astrophysical plasma.

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## Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:[10.1016/j.jqsrt.2019.02.002](https://doi.org/10.1016/j.jqsrt.2019.02.002).

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